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LETTER TO THE EDITOR

Vector coherent state theory in a group with non-commuting raising generators

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Abstract. Vector coherent state theory is used to give an explicit construction of the irreducible representations of a group with non-commuting raising generators, the simple example of the compact unitary symplectic group USp(4) ⊃ USp(2) × USp(2).

The recent generalisation of standard coherent state theory to a theory of vector coherent states (Rowe 1984, 1986, Rowe *et al* 1985; Deenen and Quesne 1984a, Quesne 1986, Castaños *et al* 1985, 1986) has proved remarkably powerful in the explicit construction of irreducible representations of a number of important groups with wide applications to various branches of physics. Introduced originally for the evaluation of matrix elements of the non-compact Sp($2d$, R) groups (Rowe *et al* 1984, Deenen and Quesne 1984b), the vector coherent state method has been applied to a large number of other groups. The compact groups USp(4) ⊃ U(1) × SU(2) and SO(8) ⊃ U(4) in their application to nuclear seniority schemes have been re-examined in terms of the vector coherent state theory (Hecht and Elliott 1985, Hecht 1985). The general case of the fermion pair algebra SO($2n$) ⊃ U(n) has recently been discussed by Rowe and Carvalho (1986). With the use of complementary Sp($2d$, R) symmetries the vector coherent state method has also been used in the construction of group theoretically sound orthonormal bases for the nuclear rotational SU(3) ⊃ SO(3) scheme (Le Blanc and Rowe 1985) and for the standard Wigner supermultiplet basis (Hecht *et al* 1987). Very recently vector coherent state theory has been used to show that the Wigner calculus for U(n) in the canonical Gel'fand U(n) ⊃ U($n-1$) chain can be reduced to an exercise in U($n-1$) recoupling, often with multiplicity-free recoupling coefficients evaluated through symmetric group techniques.

In all these examples vector coherent state theory has been applied to groups with the following general structure (Rowe 1986). The group generators can be separated into a family of commuting raising operators, A_i, their Hermitian conjugate lowering operators, B_i, and a core subalgebra, C (usually a maximal compact subalgebra) with the properties

$$B_i = A_i^+ \qquad i = 1, \ldots, m \tag{1}$$

$$[A_i, A_j] = 0 \qquad \text{for all } i, j \tag{2}$$

$$B_i|\Gamma_{\mathrm{lw}}\eta\rangle = 0 \tag{3a}$$

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or, alternatively,

$$A_i|\Gamma_{\mathrm{hw}}\eta\rangle = 0 \tag{3b}$$

where Γ_{lw} (Γ_{hw}) are the core subgroup irreducible representations containing the lowest (highest) weight states of the full group and η designates the full set of states of Γ, $\eta = 1, \ldots, D = \dim(\Gamma)$. In the vector coherent theory an algebra of the above type is mapped into a direct sum of an m-dimensional Heisenberg-Weyl algebra and an intrinsic algebra of type C.

Since there are a number of other interesting examples, such as $\mathrm{SO}(2n+1) \supset \mathrm{U}(n)$, in which all of the above criteria are satisfied, except for the commutability of the A_i, the question naturally arises: can vector coherent state theory be applied to the calculation of matrix elements of Lie algebras for which the criterion (2) is relaxed and the A_i do not form a set of commuting operators? The simplest such group is the compact unitary symplectic group $\mathrm{USp}(4) \supset \mathrm{USp}(2) \times \mathrm{USp}(2)$ generated by the commuting angular momentum operators $\boldsymbol{J}$ and $\boldsymbol{\Lambda}$, which generate the subgroup, together with the four M_J, M_Λ raising/lowering operators of type $F_{\pm 1/2, \pm 1/2}$. The notation follows that of Hecht (1965) in which the irreducible representations of USp(4) are specified by the highest weight values of J, Λ designated by $(J_m \Lambda_m)$. The aim of the present investigation is not to expand on well known results of this example. The full set of Wigner coefficients coupling an arbitrary representation $(J_m \Lambda_m)$ with the four-, five- and ten-dimensional representations are known in general algebraic form (Hecht 1965). Instead it is our aim to use this very simple example as a testing ground for the vector coherent state method. Since the quantum numbers $JM_J \Lambda M_\Lambda$ give a complete labelling of the states of $(J_m \Lambda_m)$ the K^2 matrices, which are a central feature of the vector coherent state method, are all one-dimensional, so that this is the simplest of possible examples. The group chain $\mathrm{USp}(4) \supset \mathrm{USp}(2) \times \mathrm{USp}(2)$ has recently been used (Klein and Zhang 1986) to give a boson realisation of USp(4) in very explicit analytic form. A similar programme for USp(4) has been initiated by Castaños and Moshinsky (1987). However, these investigations start with the boson mappings of three commuting raising operators of USp(4) (J_+, Λ_+ and $F_{1/2\,1/2}$ in the present notation) and thus have an aim quite different from the present one which is the construction of the states $|(J_m \Lambda_m) JM_J \Lambda M_\Lambda\rangle$ by vector coherent state techniques.

At first, vector coherent state theory does not seem to apply to this very simple example. No pair of operators from the family of $F_{\pm 1/2 \pm 1/2}$ can be found which annihilate the states of any extremal J, Λ pair for all values of M_J, M_Λ, and it appears that the vital criterion of (3) cannot be met. However, the problem can be solved by vector coherent state methods in a slightly indirect way by using the vector-valued highest weight states $|J_m M_J = J_m, \Lambda_m M_\Lambda\rangle$ which are annihilated by the raising operators $F_{1/2\,1/2} = A_1$, $F_{1/2 -1/2} = A_2$, and J_+. (The latter could have been designated A_{21}, but the name J_+ is to be preferred because of the special character of the $\boldsymbol{J}$ vector.) By lowering M_J along with J, initially keeping $M_J = J$, while generating the full spectrūm of Λ values through vector coupling techniques, the vector coherent state theory can be applied. The raising operators satisfy the commutator algebra

$$[J_+, A_1] = 0 \qquad [J_+, A_2] = 0 \qquad [A_1, A_2] = -J_+. \tag{4}$$

The commutators for the full algebra including the Hermitian conjugate lowering operators $B_1 = F_{-1/2 -1/2}$, $B_2 = F_{-1/2\,1/2}$, J_- and the core group operators J_0, Λ_+, Λ_-,

Λ_0 can be read from table 1 of Hecht (1965). The functional representation of a state vector $|\psi\rangle$ is given in terms of the complex variables $z = z_1, z_2$ and w by

$$\psi(z, w) = \sum_{M_\Lambda} \langle z, w|J_m J_m \Lambda_m M_\Lambda\rangle\langle J_m J_m \Lambda_m M_\Lambda|e^X|\psi\rangle \tag{5}$$

with

$$e^X = \exp(z_1 A_1 + z_2 A_2 + w J_+) = \exp(z_1 A_1 + z_2 A_2) \exp(w J_+). \tag{6}$$

Straightforward differentiation yields

$$\begin{aligned} \frac{\partial}{\partial z_1} e^X &= e^X (A_1 - \tfrac{1}{2} z_2 J_+) = (A_1 + \tfrac{1}{2} z_2 J_+) e^X \\ \frac{\partial}{\partial z_2} e^X &= e^X (A_2 + \tfrac{1}{2} z_1 J_+) = (A_2 - \tfrac{1}{2} z_1 J_+) e^X \\ \frac{\partial}{\partial w} e^X &= e^X J_+ = J_+ e^X \end{aligned} \tag{7}$$

where these lead to the functional representations of the raising operators

$$\begin{aligned} \Gamma(J_+) &= \frac{\partial}{\partial w} \\ \Gamma(A_1) &= \frac{\partial}{\partial z_1} + \tfrac{1}{2} z_2 \frac{\partial}{\partial w} \\ \Gamma(A_2) &= \frac{\partial}{\partial z_2} - \tfrac{1}{2} z_1 \frac{\partial}{\partial w}. \end{aligned} \tag{8a}$$

The remaining operators follow from

$$\Gamma(\mathcal{O})\psi(z, w) = \sum_{M_\Lambda} \langle z, w|J_m J_m \Lambda_m M_\Lambda\rangle\langle J_m J_m \Lambda_m M_\Lambda|(e^X \mathcal{O}\, e^{-X})\, e^X|\psi\rangle \tag{9}$$

leading to

$$\begin{aligned} \Gamma(\Lambda_+) &= \bar{\mathbf{\Lambda}}_+ + z_2 \frac{\partial}{\partial z_1} = \Lambda_+^{\text{intr}} + \Lambda_+^{\text{coll}} \\ \Gamma(\Lambda_0) &= \bar{\mathbf{\Lambda}}_0 - \tfrac{1}{2}\left(z_1 \frac{\partial}{\partial z_1} - z_2 \frac{\partial}{\partial z_2}\right) = \Lambda_0^{\text{intr}} + \Lambda_0^{\text{coll}} \\ \Gamma(\Lambda_-) &= \bar{\mathbf{\Lambda}}_- + z_1 \frac{\partial}{\partial z_2} = \Lambda_-^{\text{intr}} + \Lambda_-^{\text{coll}} \\ \Gamma(J_0) &= \bar{\boldsymbol{J}}_0 - \frac{1}{2}\left(z_1 \frac{\partial}{\partial z_1} + z_2 \frac{\partial}{\partial z_2}\right) - w \frac{\partial}{\partial w} \\ \Gamma(B_1) &= z_1(\bar{\boldsymbol{J}}_0 + \bar{\mathbf{\Lambda}}_0) - z_2 \bar{\mathbf{\Lambda}}_- - \tfrac{1}{2} z_1 \left(z_k \frac{\partial}{\partial z_k}\right) + w \frac{\partial}{\partial z_2} - \tfrac{1}{2} z_1 w \frac{\partial}{\partial w} \\ \Gamma(B_2) &= z_2(\bar{\boldsymbol{J}}_0 - \bar{\mathbf{\Lambda}}_0) - z_1 \bar{\mathbf{\Lambda}}_+ - \tfrac{1}{2} z_2 \left(z_k \frac{\partial}{\partial z_k}\right) - w \frac{\partial}{\partial z_1} - \tfrac{1}{2} z_2 w \frac{\partial}{\partial w} \\ \Gamma(J_-) &= 2w\bar{\boldsymbol{J}}_0 - w\left(z_k \frac{\partial}{\partial z_k}\right) - w^2 \frac{\partial}{\partial w} - \tfrac{1}{2} z_1^2 \bar{\mathbf{\Lambda}}_+ + \tfrac{1}{2} z_2^2 \bar{\mathbf{\Lambda}}_- - z_1 z_2 \bar{\mathbf{\Lambda}}_0 \end{aligned} \tag{8b}$$

where the summation convention is implied in $z_k\partial/\partial z_k$. The intrinsic operators $\bar{J}_0$ and $\bar{\boldsymbol{\Lambda}}=\boldsymbol{\Lambda}^{\text{intr}}$ are the operators J_0, $\boldsymbol{\Lambda}$ restricted to act only in the $(2\Lambda_m+1)$-dimensional space $|J_mJ_m\Lambda_mM_\Lambda\rangle$ of the highest weight eigenvalues $J_0=J_m$, $\Lambda=\Lambda_m$. The operators Λ^{coll} are functions only of the z_i and $\partial/\partial z_i$, e.g. $\Lambda_-^{\text{coll}}=z_1\partial/\partial z_2$. Except for the interchange of raising and lowering operators and a slightly different phase convention this is a special case of the functional representation of the $\mathrm{SO}(2n+1)\supset\mathrm{U}(n)$ algebra recently given by Rowe and Le Blanc (1986). The problem seems to have been reduced to a direct sum of a three-dimensional Heisenberg-Weyl algebra generated by $z_i, \partial/\partial z_i, w, \partial/\partial w$ and an intrinsic algebra with basis $\bar{J}_0$ and $\bar{\boldsymbol{\Lambda}}$ which generates a $\mathrm{U}(1)\times\mathrm{SU}(2)$ Lie group. However, the coupling of w and z spaces causes unnecessary complications. This can be avoided by introducing lowering operators $\mathscr{B}_1$, $\mathscr{B}_2$ which preserve the $M_J=J$ characteristic of the highest weight state

$$\mathscr{B}_1=\left(B_1-J_-A_2\frac{1}{(2J_0+1)}\right)\qquad \mathscr{B}_2=\left(B_2+J_-A_1\frac{1}{(2J_0+1)}\right). \tag{10}$$

Note that the commutator of J_+ with these $\mathscr{B}_i$ leaves only terms proportional to J_+. To convert the Dyson realisation of (8*a*) and (8*b*) to a unitary one by means of the Hermitian K operator it is useful to consider the combination $[B_1(2J_0+1)-J_-A_2]$ in place of B_1 leading to

$$\begin{aligned}
&K^{-1}\Gamma(B_1(2J_0+1)-J_-A_2)K\\
&\quad=\gamma(B_1(2J_0+1)-J_-A_2)\\
&\quad=(\gamma((2J_0+1)A_1-B_2J_+))^\dagger=(K^{-1}\Gamma((2J_0+1)A_1-B_2J_+)K)^\dagger\\
&\quad=\left(K^{-1}\left(2\bar{J}_0+1-z_k\frac{\partial}{\partial z_k}-2w\frac{\partial}{\partial w}\right)\left(\frac{\partial}{\partial z_1}+\frac{z_2}{2}\frac{\partial}{\partial w}\right)K\right)^\dagger-\left(K^{-1}\Gamma(B_2)\frac{\partial}{\partial w}K\right)^\dagger\\
&\quad=Kz_1\left((2\bar{J}_0+1)-z_1\frac{\partial}{\partial z_1}-z_2\frac{\partial}{\partial z_2}\right)K^{-1}
\end{aligned} \tag{11}$$

where the last step is valid when these operators are permitted to act only between states with $M_J=J$ so that operators $\partial/\partial w$ acting to the right and w acting to the left yield zero and can be omitted. Equation (11) can then be put into the form

$$\begin{aligned}
&\left[(2J_0+2)\left(z_1(\bar{J}_0+\bar{\Lambda}_0)-z_2\bar{\Lambda}_--\tfrac{1}{2}z_1z_k\frac{\partial}{\partial z_k}\right)-(-\tfrac{1}{2}z_1^2\bar{\Lambda}_++\tfrac{1}{2}z_2^2\bar{\Lambda}_--z_1z_2\bar{\Lambda}_0)\frac{\partial}{\partial z_2}\right]K^2\\
&\quad=K^2z_1(2J_0+1)
\end{aligned} \tag{12}$$

and, similarly from $[B_2(2J_0+1)+J_-A_1]$,

$$\begin{aligned}
&\left[(2J_0+2)\left(z_2(\bar{J}_0-\bar{\Lambda}_0)-z_1\bar{\Lambda}_+-\tfrac{1}{2}z_2z_k\frac{\partial}{\partial z_k}\right)+(-\tfrac{1}{2}z_1^2\bar{\Lambda}_++\tfrac{1}{2}z_2^2\bar{\Lambda}_--z_1z_2\bar{\Lambda}_0)\frac{\partial}{\partial z_1}\right]K^2\\
&\quad=K^2z_2(2J_0+1).
\end{aligned} \tag{13}$$

The first term in the left-hand side of (12) and (13) can be simplified through an Ω operator which is part of the standard machinery of vector coherent state theory

$$\Omega=-\frac{1}{4}\left(z_k\frac{\partial}{\partial z_k}\right)\left(z_k\frac{\partial}{\partial z_k}\right)+(\bar{J}_0+\tfrac{1}{4})\left(z_k\frac{\partial}{\partial z_k}\right)-\boldsymbol{\Lambda}^2+\boldsymbol{\Lambda}^2_{\text{coll}}+\boldsymbol{\Lambda}^2_{\text{intr}}. \tag{14}$$

The second term is best expressed through a Λ space vector-coupled form, noting that $(z_1, z_2) \equiv (z_{-1/2-1/2}, z_{-1/2+1/2})$, so that (12) and (13) can be put into the form

$$\left((2J_0+2)[\Omega, z_1] - \sqrt{3}[\bar{\mathbf{\Lambda}}^1 \times (z^2)^1]^0 \frac{\partial}{\partial z_2}\right) K^2 = K^2 z_1(2J_0+1) \tag{15}$$

$$\left((2J_0+2)[\Omega, z_2] + \sqrt{3}[\bar{\mathbf{\Lambda}}^1 \times (z^2)^1]^0 \frac{\partial}{\partial z_1}\right) K^2 = K^2 z_2(2J_0+1). \tag{16}$$

Although no further simplification has been achieved of the second terms in these equations it is easy to show that these second terms have matrix elements proportional to z_i. In a Λ space vector-coupled basis $|J_m J = M_J, [\Lambda_m \times \Lambda_{\text{coll}}]\Lambda M_\Lambda\rangle$ with the intrinsic Λ spin, Λ_m, coupled with a collective Λ angular momentum with Λ space collective functions $(\Lambda_{\text{coll}} \equiv \Lambda_c)$,

$$\frac{z_1^{\Lambda_c - M_c}}{[(\Lambda_c - M_c)!]^{1/2}} \frac{z_2^{\Lambda_c + M_c}}{[(\Lambda_c + M_c)!]^{1/2}} = \mathscr{Y}_{\Lambda_c M_c}(z) \tag{17}$$

we note that $\Lambda_c = 1/2(n_1 + n_2)$, where

$$\left(z_1 \frac{\partial}{\partial z_1} + z_2 \frac{\partial}{\partial z_2}\right) \mathscr{Y}_{\Lambda_c M_c} = (n_1 + n_2)\mathscr{Y}_{\Lambda_c M_c} = 2\Lambda_c \mathscr{Y}_{\Lambda_c M_c}. \tag{18}$$

In the subspace of states with $M_J = J$, $J = J_m - \frac{1}{2}(n_1 + n_2)$, and the eigenvalue of $w\,\partial/\partial w$ has the value $n_w = 0$, so that

$$\Lambda_c = J_m - J. \tag{19}$$

Equation (17) also yields the z space reduced matrix elements

$$\langle \Lambda_c + \tfrac{1}{2} \| z \| \Lambda_c \rangle = (2\Lambda_c + 1)^{1/2} = -\langle \Lambda_c - \tfrac{1}{2} \| \partial/\partial z \| \Lambda_c \rangle \tag{20}$$

$$\langle \Lambda_c + 1 \| z^2 \| \Lambda_c \rangle = [(2\Lambda_c + 1)(\Lambda_c + 1)]^{1/2} \qquad \text{also} \qquad \langle \Lambda_m \| \mathbf{\Lambda} \| \Lambda_m \rangle = [\Lambda_m(\Lambda_m + 1)]^{1/2}.$$

Finally, with

$$\langle [\Lambda_m \times (\Lambda_c + \tfrac{1}{2})]\Lambda' \| z \| [\Lambda_m \times \Lambda_c]\Lambda \rangle = U(\Lambda_m \Lambda_c \Lambda' \tfrac{1}{2}; \Lambda \Lambda_c + \tfrac{1}{2})[(2\Lambda_c + 1)]^{1/2} \tag{21}$$

$$\langle [\Lambda_m \times (\Lambda_c - \tfrac{1}{2})]\Lambda' \| \partial/\partial z \| [\Lambda_m \times \Lambda_c]\Lambda \rangle = -U(\Lambda_m \Lambda_c \Lambda' \tfrac{1}{2}; \Lambda \Lambda_c - \tfrac{1}{2})[(2\Lambda_c + 1)]^{1/2} \tag{22}$$

$$\begin{aligned}&\sqrt{3}\langle [\Lambda_m \times (\Lambda_c + 1)]\Lambda \| [\bar{\mathbf{\Lambda}}^1 \times (z^2)^1]^0 \| [\Lambda_m \times \Lambda_c]\Lambda \rangle \\ &\quad = -U(\Lambda_m 1 \Lambda \Lambda_c; \Lambda_m \Lambda_c + 1)[\Lambda_m(\Lambda_m + 1)(2\Lambda_c + 1)(\Lambda_c + 1)]^{1/2}\end{aligned} \tag{23}$$

and, using explicit expressions for the well known Racah coefficients,

$$\begin{aligned}&\langle J_m J - \tfrac{1}{2} = M'_J, [\Lambda_m \times (\Lambda_c + \tfrac{1}{2})]\Lambda' \| \sqrt{3}[\bar{\mathbf{\Lambda}}^1 \times (z^2)^1]^0\, \partial/\partial z \| J_m J = M_J [\Lambda_m \times \Lambda_c]\Lambda \rangle \\ &\quad = \begin{Bmatrix} -\tfrac{1}{2}(\Lambda_m + \Lambda_c - \Lambda)(\Lambda_m + \Lambda - \Lambda_c + 1) \\ \tfrac{1}{2}(\Lambda_m + \Lambda_c + \Lambda + 1)(\Lambda_c + \Lambda - \Lambda_m) \end{Bmatrix} \\ &\quad \times \langle J_m J - \tfrac{1}{2}[\Lambda_m \times (\Lambda_c + \tfrac{1}{2})]\Lambda' \| z \| J_m J [\Lambda_m \times \Lambda_c]\Lambda \rangle\end{aligned} \tag{24}$$

where the upper and lower equation applies to $\Lambda' = (\Lambda + \frac{1}{2})$ and $(\Lambda - \frac{1}{2})$, respectively. The needed difference of Ω eigenvalues $\langle \Omega \rangle_{J\Lambda}$ follows at once from (14):

$$\langle \Omega \rangle_{(J-1/2)(\Lambda \pm 1/2)} - \langle \Omega \rangle_{J\Lambda} = \begin{Bmatrix} J - \Lambda + \Lambda_c \\ J + \Lambda_c + \Lambda + 1 \end{Bmatrix} \qquad \begin{matrix} \Lambda' = \Lambda + \frac{1}{2} \\ \Lambda' = \Lambda - \frac{1}{2}. \end{matrix} \tag{25}$$

With these results and $\Lambda_c = J_m - J$, (15) and (16) lead to

$$\frac{K^2(J-\frac{1}{2}, \Lambda'=\Lambda\pm\frac{1}{2})}{K^2(J,\Lambda)}(2J+1)$$

$$= \begin{Bmatrix} \frac{1}{2}(J_m+\Lambda_m+J-\Lambda+1)(J_m-\Lambda_m+J-\Lambda) \\ \frac{1}{2}(J_m+\Lambda_m+J+\Lambda+2)(J_m-\Lambda_m+J+\Lambda+1) \end{Bmatrix}. \tag{26}$$

Since all K^2 matrices are one dimensional, vector coherent state theory at once leads to the reduced matrix elements of the lowering/raising generators. It will be convenient to use matrix elements reduced in both J and Λ angular momenta. With the explicit value of $\langle JJ\frac{1}{2}-\frac{1}{2}|J-\frac{1}{2}J-\frac{1}{2}\rangle = [2J/(2J+1)]^{1/2}$ vector coherent state theory gives

$$\langle (J_m\Lambda_m)J-\tfrac{1}{2}\Lambda'\|\boldsymbol{B}\|(J_m\Lambda_m)J\Lambda\rangle\left(\frac{2J}{(2J+1)}\right)^{1/2}$$

$$= \frac{K(J-\frac{1}{2},\Lambda')}{K(J,\Lambda)}\, U(\Lambda_m\Lambda_c\Lambda'\tfrac{1}{2};\, \Lambda\Lambda_c+\tfrac{1}{2})\langle \Lambda_c+\tfrac{1}{2}\|z\|\Lambda_c\rangle. \tag{27}$$

Iteration of (26) yields

$$K^2(n_1, n_2) = \frac{(2J_m+2\Lambda_m+2)!(2J_m-2\Lambda_m)!(2J_m+1)!(2J_m-n_1-n_2+1)!}{2^{n_1+n_2}(2J_m+2\Lambda_m-n_1+2)!(2J_m-2\Lambda_m-n_2)!(2J_m+1-n_2)!(2J_m+1-n_1)!} \tag{28}$$

with

$$J = J_m - \tfrac{1}{2}n_1 - \tfrac{1}{2}n_2 \qquad \Lambda = \Lambda_m - \tfrac{1}{2}n_1 + \tfrac{1}{2}n_2 \tag{29}$$

where the limits

$$0 \leqslant n_2 \leqslant 2(J_m - \Lambda_m) \qquad 0 \leqslant n_1 \leqslant 2\Lambda_m \tag{30}$$

follow for n_2 from the structure of K^2 and for n_1 from the value of the Racah coefficient.

Although the group $\mathrm{USp}(4) \supset \mathrm{USp}(2)\times\mathrm{USp}(2)$ is an extremely simple example of a group with non-commuting raising generators, it may point the way toward a solution for more complicated cases. The key to the successful use of the vector coherent state method may be the effective uncoupling of the two types of bosonic variables, such as w and z. The existence of the additional true quantum numbers J makes this particularly simple in the present example, and more complicated cases may require more laborious techniques, such as matrix diagonalisation. Nevertheless, the simple example of $\mathrm{USp}(4) \supset \mathrm{USp}(2)\times\mathrm{USp}(2)$ gives some indication that the vector coherent state method may be useful in the construction of irreducible representations for groups with non-commuting raising generators.

An alternate approach used in the non-compact $\mathrm{Sp}(4, R)$ version of this group (Castaños and Moshinsky 1987) starts with a general expression for the overlap kernel developed in terms of three commuting raising generators. This is then transformed into the appropriate basis on expansion. Although less direct than the present method it may prove powerful in more general cases.

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